Markscheme

May 2024

Mathematics:
applications and interpretation

Standard level

Paper 1

Instructions to Examiners

Abbreviations

M   Marks awarded for attempting to use a correct **Method**.

A   Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.

R   Marks awarded for clear **Reasoning**.

AG   Answer given in the question and so no marks are awarded.

FT   Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1   General

Award marks using the annotations as noted in the markscheme eg **M1**, **A2**.

2   Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.

- It is generally not possible to award **M0** followed by **A1**, as **A** mark(s) depend on the preceding **M** mark(s), if any.

- Where **M** and **A** marks are noted on the same line, *e.g. M1A1*, this usually means **M1** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and **A1** for using the **correct** values.

- Where there are two or more **A** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award **A0A1A1**.

- Where the markscheme specifies **A3**, **M2** *etc*., do **not** split the marks, unless there is a note.

- The response to a "show that" question does not need to restate the **AG** line, unless a **Note** makes this explicit in the markscheme.

- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.

- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award **FT** marks as appropriate but do not award the final **A1** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... (incorrect decimal value)	No. Last part in question.	Award **A1** for the final mark (condone the incorrect further working)
2.	$\dfrac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award **A0** for the final mark (and full **FT** is available in subsequent parts)

3   Implied marks

Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4   Follow through marks (only applied after an error is made)

Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then **FT** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is **(M1)A1**, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further **A** marks can be awarded for work which uses the error, but **M** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.

- If the candidate's answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any **FT** marks in the subsequent parts. This includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these **FT** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 **Mis-read**

If a candidate incorrectly copies values or information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread and do not award the first mark, even if this is an **M** mark, but award all others as appropriate.

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.

- If the **MR** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.

- **MR** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 **Alternative methods**

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 **Alternative forms**

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1,9$ or 1000 and $1,000$ and 1.000.

- Do not accept final answers written using calculator notation. However, **M** marks and intermediate **A** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8   Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an *A* mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\dfrac{25}{4}}$ should be written as $\dfrac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\dfrac{10}{4}$ may be left in this form or written as $\dfrac{5}{2}$. However, $\dfrac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate *A* marks do NOT need to be simplified.

9   Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10.   Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) identifying the largest and smallest values: ($)255, ($)49   **(M1)**
($)206   **A1**

[2 marks]

(b) (i) ($)137 (137.1)   **(M1)A1**

(ii) ($)74.5 (74.4693…)   **A1**

> **Note:** The **(M1)** mark is for correct GDC use and hence can be awarded if either of the values is correct. An answer of 78.4976… in (b)(ii) is awarded **A0** but is sufficient to credit the **(M1)**.

[3 marks]

(c) (i) (mean =) ($)117 (117.1)   **A1**

(ii) (standard deviation =) ($)74.5 (74.4693…)   **A1**

> **Note:** If their answer to part (c)(ii) is incorrect, it should match their answer to part (b)(ii) to be awarded **A1(FT)**.

[2 marks]
[Total: 7 marks]

2. (a) attempt to substitute into cosine rule formula   **(M1)**
$(BC^2 =)\ 15^2 + 25^2 - 2 \times 15 \times 25 \times \cos(22)$ **(A1)**
$(BC =)\ 12.4\ (cm)\ (12.4343…)$ **A1**

[3 marks]

(b) selecting sine rule formula **OR** cosine rule formula   **(M1)**
$$\frac{12.4343…}{\sin 22} = \frac{15}{\sin A\hat{B}C} \quad \textbf{OR} \quad \left(\cos A\hat{B}C =\right)\frac{25^2 + 12.4343…^2 - 15^2}{2 \times 25 \times 12.4343…}$$ **(A1)**

> **Note:** Award **M1A1** for correct cosine rule formula to find $A\hat{B}C$.

$(A\hat{B}C =)\ 26.9°\ (26.8658…°)$ **A1**

> **Note:** Accept 26.9461… from use of 12.4 in the sine rule formula and 26.7267… in the cosine rule formula.

[3 marks]
[Total: 6 marks]

3. (a) H_0: the size of peppers is independent of colour
H_1: the size of peppers is not independent of colour **A1**

> **Note:** Award **A1** for both hypotheses correct. Accept "not associated" in place of independent. Do not accept "correlated" or "related" or "affected".

[1 mark]

(b) (i) $\chi^2_{\text{calc}} = 22.5$ $(22.5483...)$ **A2**

(ii) $22.5483... > 9.49$ **OR** $0.000155837... < 0.05$ **R1**
(there is sufficient evidence to) reject the null hypothesis **A1**

> **Note:** Do not award **R0A1**.
> Accept "accept the alternative hypothesis".
> Their conclusion must be consistent with their χ^2_{calc} (or p-value) and their hypothesis.
> Accept $\chi^2_{\text{calc}} > \chi^2_{\text{crit}}$ or $p <$ sig level provided their χ^2_{calc} value or p-value is seen.

[4 marks]
[Total: 5 marks]

4. (a) expressing one variable in terms of the other **(M1)**
$n = kB$ **OR** $n \propto B$
$60 = 5k$
$k = 12$ **(A1)**
$n = 12(7)$
$= 84$ **A1**

[3 marks]

(b) expressing t terms of reciprocal of B **(M1)**
$t = \dfrac{c}{B}$ **OR** $t \propto \dfrac{1}{B}$

$8 = \dfrac{c}{5}$

$c = 40$ **(A1)**

$(t =) \dfrac{40}{12}$ $\left(\dfrac{10}{3},\ 3.33,\ 3.33333... \right)$ (hours) **OR** 3 hours 20 minutes **A1**

> **Note:** Award at most **M1A1A0** for the use of $t = cB$ in part (b) if $n = \dfrac{k}{B}$ was seen and penalized in part (a).

[3 marks]
[Total: 6 marks]

5.   (a)   **EITHER**

N = 72   N = 6
I = 1.25   I = 1.25
PV = −3000   **OR**   PV = −3000
P/Y = 12   P/Y = 1
C/Y = 12   C/Y = 12   *(M1)(A1)*

Note: Award *M1* for attempt to use finance app on GDC, *A1* for all entries correct.

OR

$$3000\left(1+\frac{1.25}{1200}\right)^{72}$$ *(M1)(A1)*

Note: Award *M1* for using CI formula, *A1* for correct entries.

THEN

FV = (\$) 3233.53   *A1*

Note: Answer must be correct to 2 dp for the final *A1* to be awarded.

[3 marks]

(b)   **EITHER**

I = 1.25   I = 1.25
PV = −3000   PV = −3000
FV = 3550   **OR**   FV = 3550
P/Y = 12   P/Y = 1
C/Y = 12   C/Y = 12   *(A1)*

Note: Award *A1* for all entries correct and opposite signs for PV and FV values.

OR

$$3550 = 3000\left(1+\frac{1.25}{1200}\right)^{12N}$$ *(A1)*

Note: Award *A1* for all entries correct.

THEN

N = 162 (161.686…) (months)   *A1*

[2 marks]

(c)   18450   *A1*

[1 mark]

(d)   N = 96   N = 96
       I = 12.6   I = 1.05
       PV = −18450   **OR**   PV = −18450
       FV = 0   FV = 0
       P/Y = 12   P/Y = 1
       C/Y = 12   C/Y = 1   **(M1)(A1)**

> **Note:** Award **M1** for an attempt to use finance app on GDC, **A1** for $N = 96$,
> $PV = −18450$ and $FV = 0$,

       PMT = ($) 306   **A1**

> **Note:** The answer must be correct to the nearest dollar for the final **A1** to be awarded.

                                                                        [3 marks]
                                                                  [Total: 9 marks]

6.   (a)   (i)   recognition of binomial distribution (condone incorrect parameter)   **(M1)**
                     e.g. $M \sim B(20, 0.04)$ **OR** $P(M = 2) = \text{binpdf}(20, 0.04, 2)$
                     $= 0.146 \ (0.145799...)$ **A1**

             (ii)   recognition the cumulative probability required   **(M1)**
                     e.g. $P(M \geq 3) = 1 - \text{bincdf}(20, 0.04, 2)$ **OR** $\text{bincdf}(20, 0.04, 3, 20)$
                     $= 0.0439 \ (0.0438627...)$ **A1**

                                                                        [4 marks]

       (b)   either one of two terms in expected value formula correct   **(M1)**
             $50(20(0.96)) + 15(20(0.04))$
             $= 972$ (pesos)   **A1**

                                                                        [2 marks]
                                                                  [Total: 6 marks]

7.   (a)   correct substitution of 0.0003 into the formula   **(A1)**

$$pH = -\log_{10}(0.0003)$$

$$= 3.52 \quad (3.52287\ldots)$$ **A1**

[2 marks]

(b)   **EITHER**
attempt to change to exponential form   **(M1)**

$$\left[H^+\right] = 10^{-6.6}$$

OR

attempt to solve $6.6 = -\log_{10}\left[H^+\right]$ **(M1)**

(graphically or using numerical solver)
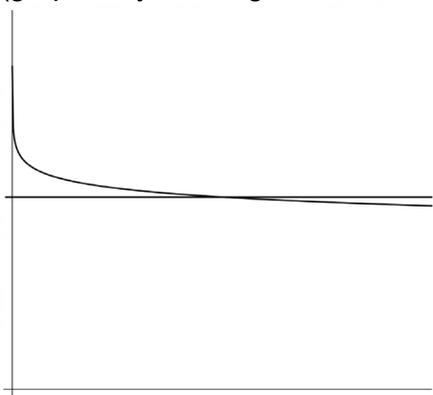
THEN

$\left[H^+\right] = 0.000000251$ (moles per litre) $(0.000000251188\ldots, 2.51\times10^{-7})$ **A1**

Note: Award **M1A0** for an answer of 2.51 (2.51188...) seen. For an attempt to solve $6.6 = -\log_{10}\left[H^+\right]$, award **M0A0** if a substitution of 6.6 into the formula is seen without an answer or some indication of using numerical solver.

[2 marks]

(c) $2 = -\log_{10}\left[H^+\right]$, $4.5 = -\log_{10}\left[H^+\right]$

10^{-2} (0.01)   **OR** $10^{-4.5}$ (0.0000316227...)   **(A1)**

substitution of their values into correct ratio   **(M1)**

$$\frac{10^{-2}}{10^{-4.5}} \quad \textbf{OR} \quad \frac{0.01}{0.0000316227\ldots}$$

$$= 316.227\ldots = 316$$ **A1**

Note: Some candidates may subtract logs and hence look to solve $\log_{10}[H^+] = 2.5$.

[3 marks]
[Total: 7 marks]

8. (a) recognizing that only way to score 7 is to achieve a head and a 6 on die   *(M1)*

e.g. $\dfrac{1}{6}$ and $\dfrac{1}{2}$ seen in an attempt to combine probabilities

$\left(\dfrac{1}{6}\times\dfrac{1}{2}=\right)\dfrac{1}{12}$ $(0.0833333...)$ *A1*

> **Note:** Accept 0.0835 from the use of 0.167.

[2 marks]

(b) there are two ways to score (e.g.) 5
achieve a head and a 4 on die, or a tail and a 5 on die   *(M1)*

$\left(2\left(\dfrac{1}{6}\times\dfrac{1}{2}\right)=\right)\dfrac{2}{12}\ \left(\dfrac{1}{6},\,0.167,\,0.16666...\right)$ *A1*

> **Note:** Award these marks for equivalent working for the 2, 3, 4 or 6 point scenarios.

Final Score	1	2	3	4	5	6	7
Probability	$\dfrac{1}{12}$	$\dfrac{1}{6}$	$\dfrac{1}{6}$	$\dfrac{1}{6}$	$\dfrac{1}{6}$	$\dfrac{1}{6}$	$\dfrac{1}{12}$

A1

> **Note:** Award *A1* for a completely correct table. Award at most *(M1)A1A0* if their follow-through answer from part (a) leads to a total probability not equal to 1.

[3 marks]

(c) **EITHER**
multiplying at least two columns from their table   *(M1)*

$1\times\dfrac{1}{12}+2\times\dfrac{1}{6}+...+6\times\dfrac{1}{6}+7\times\dfrac{1}{12}$

OR
recognizing the probabilities in the table are symmetric   *(M1)*

OR *(for HL markscheme and/or removed from SL at publication)*
Considering the sum of two random variables   *(M1)*
$\mathrm{E}(X+Y)=\mathrm{E}(X)+\mathrm{E}(Y)\ \left(=3.5+0.5\right)$

THEN
(expected value $=$) 4   *A1*

> **Note:** Accept 4.01 $(4.00640…)$ from use of their 3 sf values from (b).
> Award at most *M1A0* if their final answer is not in the range $1-7$

[2 marks]
[Total: 7 marks]

9. **EITHER**
recognizing that Eefje's is an AP **OR** that Shumay's is a GP   **(M1)**

$42.195 = 5 + (n-1)(2)$ **(A1)**
$n_E = 19.6 \ (19.5975\ldots)$ (accept 20)   **A1**

$42.195 = 5(1.13)^{n-1}$ **(A1)**
$n_S = 18.5 \ (18.4513\ldots)$ (accept 19)   **A1**

OR   (Graphical method)

e.g

$(18.451, 42.195)$

$(19.598, 42.195)$

OR

$(18.451, 0)$

$(19.598, 0)$
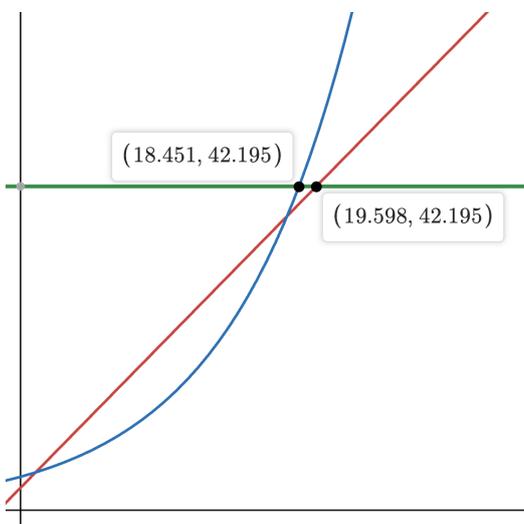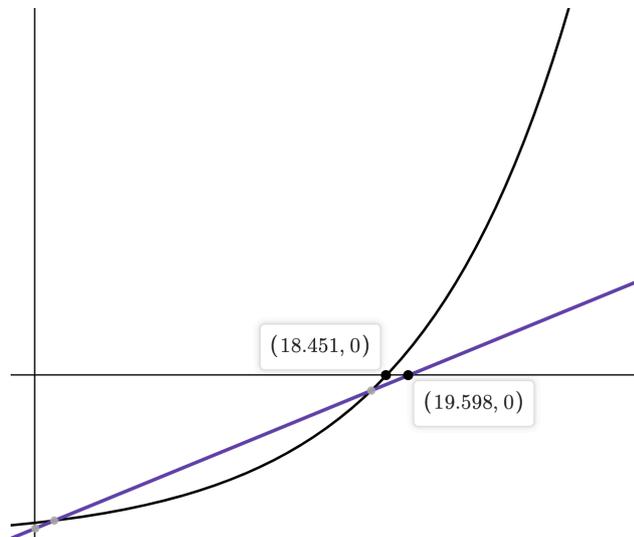
Sketching a straight line (Eefje's)   **OR**   an exponential curve (Shumay's)   **(M1)**

Note: The horizontal line (marathon length) does not need to be seen to be awarded the **M1**.

Horizontal line (marathon length) **and** Eefje's linear drawn   **(A1)**
x-coordinate of the point of intersection 19.6 (19.5975....)   **A1**

Horizontal line (marathon length) **and** Shumay's curved drawn   **(A1)**
x-coordinate of the point of intersection 18.5 (18.4513....)   **A1**

Note: A ruler does not need to be used for the lines.

THEN
so Shumay (will be the first to run the distance of a marathon) <u>on day 19</u>   **A1**

> **Note:** Award the final **A** mark for a correctly rounded-up integer value. **FT** from their n_E
>
> and n_s values. Award **A0** for a final result of "Shumay, day 19" if it is clear that
>
> Eefje/the AP has not been considered (either by finding the 19.5975, or by
> substituting $n = 19$ into AP and showing distance is less than marathon).
> Accept lists showing recognition that Eefje's is an AP **OR** that Shumay's is a GP
> **If trial and error method is used:**
> Award **A1A1** for finding the distance for Eefje on day $20 = 43$ and **A1A1** for
> finding the distance for Shumay on day $19 = 45.12$.

[Total: 6 marks]

10. 3 (seen at any stage of their work)   *(A1)*

$$\frac{dy}{dx} = 2ax + b$$ *(M1)(A1)*

> **Note:** Award **M1** for an attempt to find $\dfrac{dy}{dx}$.

substituting $x = 2$ and their gradient into their derivative   *(M1)*
$3 = 2a(2) + b$
substituting $(2, 4)$ into original equation   *(M1)*
$4 = a(2)^2 + b(2) - 10$
solving equations $a = -2$
                    $b = 11$ *A1A1*

> **Note:** Using $m = -\dfrac{1}{3}$, gives $a = -\dfrac{11}{3}, b = \dfrac{43}{3}$, award **A0M1A1M1M1A0A1**.
>
> For the final **A** marks, award at most **A1A0** if correct values are
> unlabelled or incorrectly labelled.

[Total: 7 marks]

11. attempt to substitute into area of triangle formula **(M1)**
(sheep's field area =) $0.5 \times 15 \times 21 \times \sin(78°)$

$= 154.058\ldots \ (\mathrm{m}^2)$ **A1**

EITHER

(goat's field area =) $\dfrac{282}{360} \times \pi \times 8^2$ **(A1)(A1)**

> **Note:** Award **A1** for 282, **A1** for correct entries in formula (including their 282).

OR

$\pi \times 8^2 - \dfrac{78}{360} \times \pi \times 8^2$ **(M1)(A1)**

> **Note:** Award **A1** for minor sector area, **M1** for subtracting their sector area from circle area.

THEN

$= 157.498\ldots \left(\dfrac{752\pi}{15} \right) \ (\mathrm{m}^2)$ **A1**

the goat has most area by $3.44 \ (\mathrm{m}^2) \ (3.44026\ldots)$ **A1**

> **Note:** Accept 154 and 157 for the intermediate **A1** marks, but do NOT follow
> through within the question; a final answer of $3 \ \mathrm{m}^2$ is awarded **A0**.

[Total: 6 marks]

12. (a) $\pi x + 2y \ (= 20)$ **A1**

> **Note:** Award **A0** for an unsimplified answer.

[1 mark]

(b) attempt to combine area formulas to express A in terms of x and y **M1**

$$A = xy + \frac{\pi x^2}{4}$$

$$y = \frac{20 - \pi x}{2} \ \left(= 10 - \frac{\pi x}{2}\right)$$ **A1**

correct substitution for y and expansion of brackets **A1**

$$A = x\left(\frac{20 - \pi x}{2}\right) + \frac{\pi x^2}{4} = \frac{20x - \pi x^2}{2} + \frac{\pi x^2}{4}$$

$$= 10x - \frac{\pi x^2}{4}$$ **AG**

> **Note:** The **AG** line must be stated for the final **A1** to be awarded.

[3 marks]

(c) attempt at power rule e.g. one correct term **(M1)**

$$10 - \frac{\pi x}{2}$$ **A1**

[2 marks]

(d) **EITHER**
setting their derivative to zero **M1**

$$10 - \frac{\pi x}{2} = 0 \quad \textbf{OR} \quad \frac{\mathrm{d}A}{\mathrm{d}x} = 0$$

OR
attempt at finding the root of the graph of their derivative **M1**
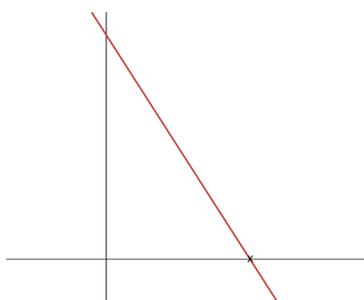
THEN

$$x = \frac{20}{\pi} \ \text{(m)}$$ **A1**

(given that function is a negative quadratic, only stationary point is a maximum)

> **Note:** Do not award the M mark if their derivative is not used to find the x-value.
> Award at most **M1A0** if their answer is not given in exact form.

[2 marks]
[Total: 8 marks]